ENGINEERING MECHANICS - STATICS
CHAPTER-3

[EQUILIBRIUM]

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3/1 INTRODUCTION

Statics deals primarily with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures. We will use the concepts of developed in Chapter 2 involving forces, moments, couples, and resultants as we apply the principles of equilibrium.

When a body is in equilibrium, the resultant of *all* forces acting on it is zero. Thus, the resultant force **R** and the resultant couple **M** are both zero, and we have the equilibrium equations

$$\mathbf{R} = \Sigma \mathbf{F} = 0 \qquad \mathbf{M} = \Sigma \mathbf{M} = 0 \qquad (3/1)$$

All physical bodies are three-dimensional, but we can treat many of them as two-dimensional.

3/2 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

A *mechanical system* is defined as a body or group of bodies which can be conceptually isolated from all other bodies. The bodies may be rigid or nonrigid. In statics we study primarily forces which act on rigid bodies at rest, although we also study forces acting on fluids in equilibrium.

To analyze forces acting on a body, it is essential that we *isolate* the body in question from all other bodies so that a complete and accurate account of all forces acting on this body can be taken. This *isolation* should exist mentally and should be represented on paper. The diagram of such an isolated body with the representation of *all* external forces acting *on* it is called a ***free-body diagram***.

Construction of Free-Body Diagrams

Step 1. Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

Step 2. Next isolate the chosen system by drawing a diagram which represents its *complete external boundary*.

Step 3. Identify all forces which act *on* the isolated system as applied *by* the removed contacting and attracting bodies.

Step 4. Show the choice of coordinate axes directly on the diagram. Pertinent dimensions may also be represented for convenience.

Modeling the Action of Forces

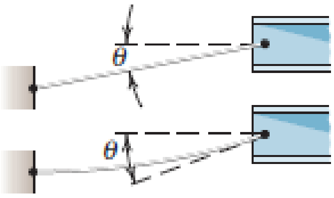
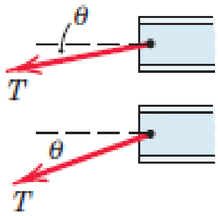
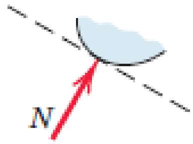
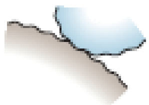
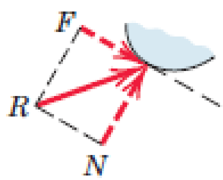
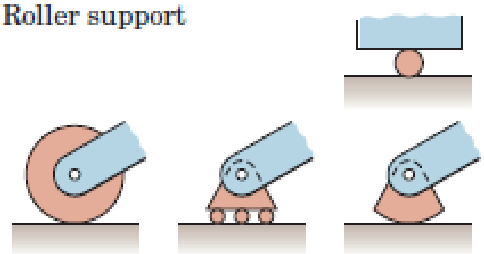
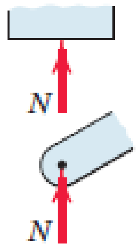
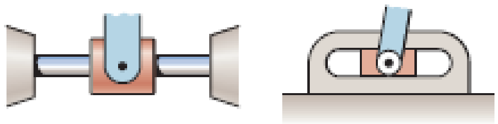
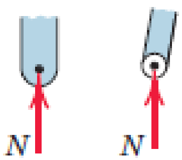
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent *F* (frictional force) as well as a normal component *N* of the resultant
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

Figure 3/1

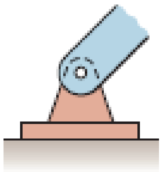
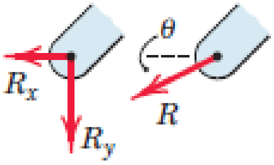
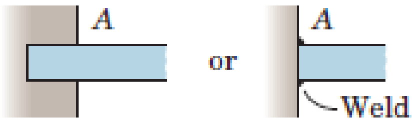
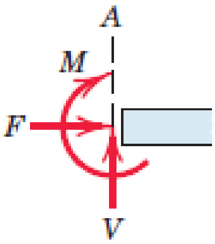
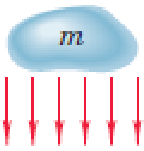
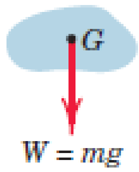
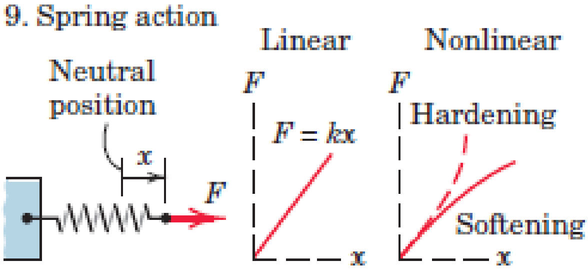
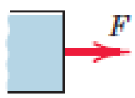
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Figure 3/1, continued

Examples of Free-Body Diagrams

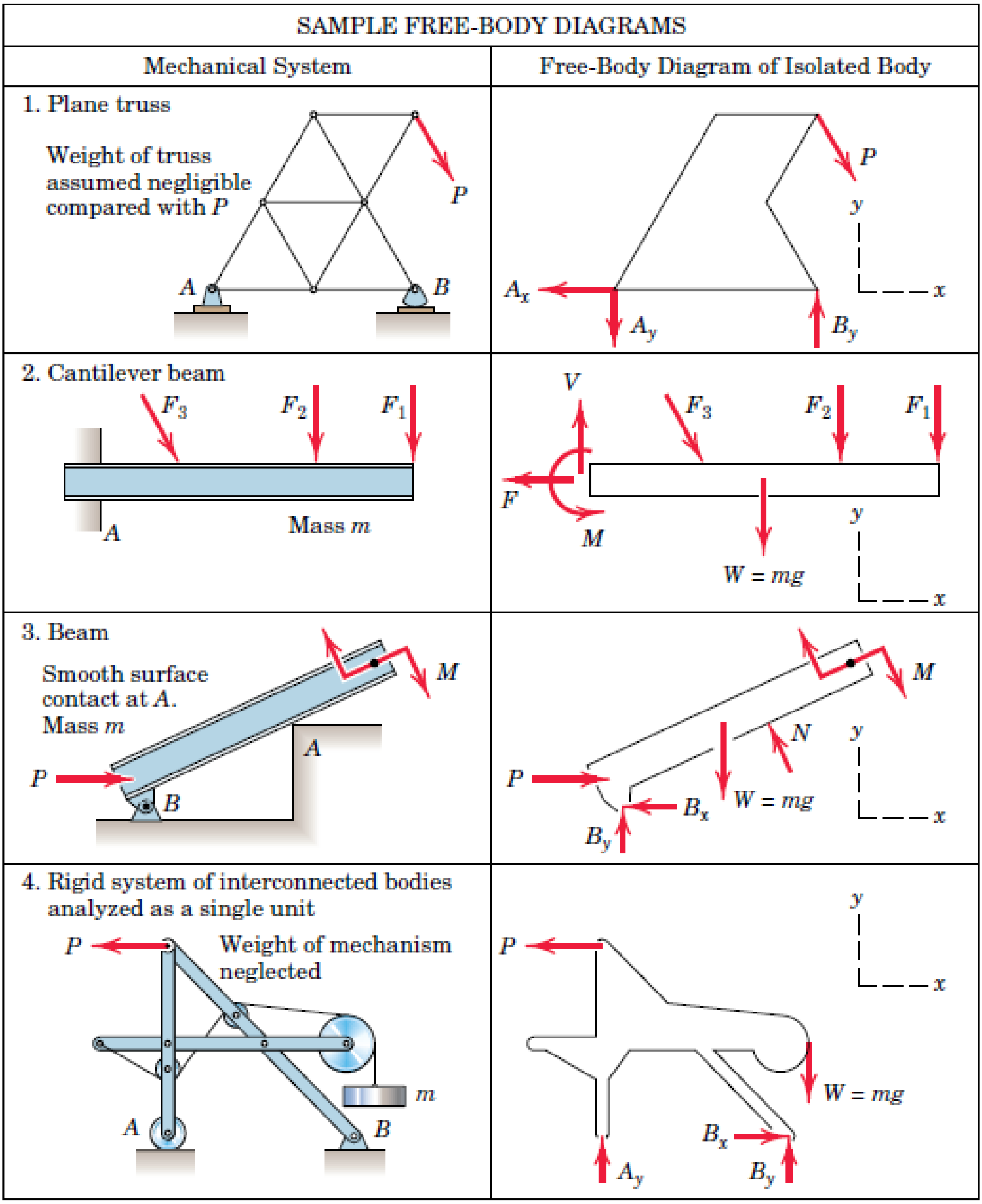


Figure 3/2

FREE-BODY DIAGRAM EXERCISES

3/A In each of the five following examples, the body to be isolated is shown in the left-hand diagram, and an *incomplete* free-body diagram (FBD) of the isolated body is shown on the right. Add whatever forces are necessary in each case to form a complete free-body diagram. The weights of the bodies are negligible unless otherwise indicated. Dimensions and numerical values are omitted for simplicity.

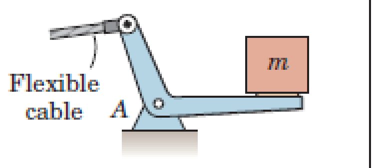
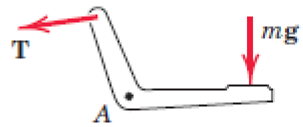
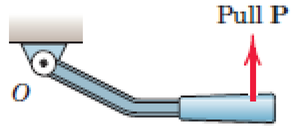
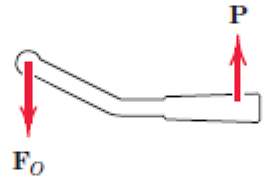
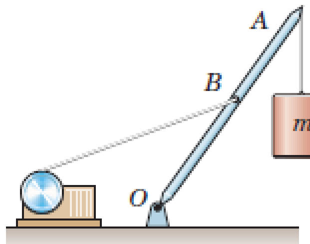
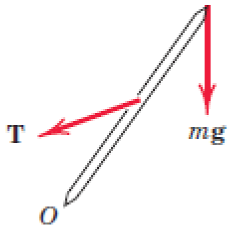
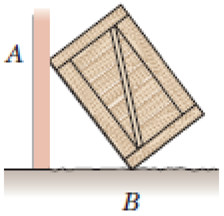
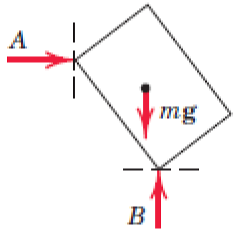
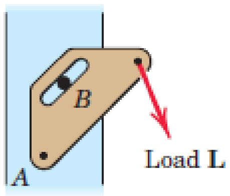
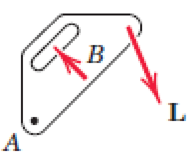
	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

Figure 3/A

3/B In each of the five following examples, the body to be isolated is shown in the left-hand diagram, and either a *wrong* or an *incomplete* free-body diagram (FBD) is shown on the right. Make whatever changes or additions are necessary in each case to form a correct and complete free-body diagram. The weights of the bodies are negligible unless otherwise indicated. Dimensions and numerical values are omitted for simplicity.

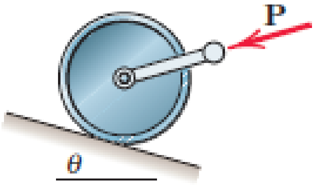
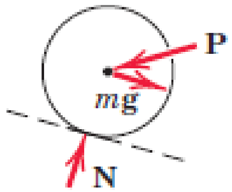
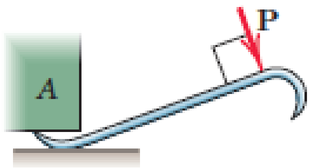
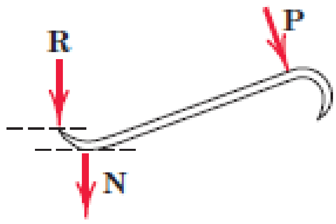
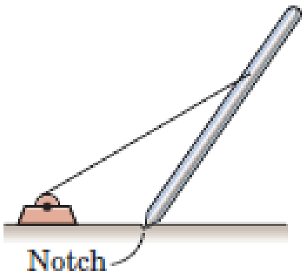
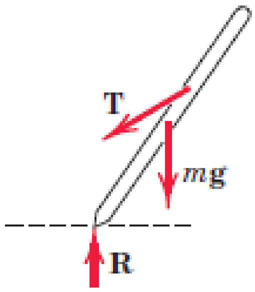
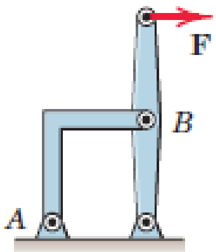
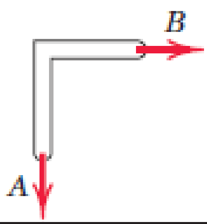
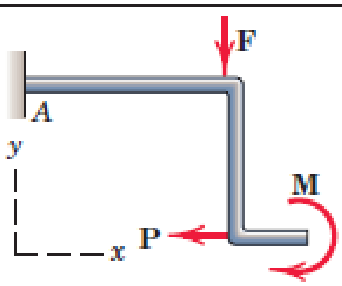
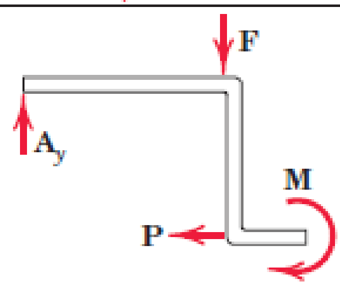
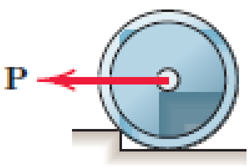
	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

Figure 3/B

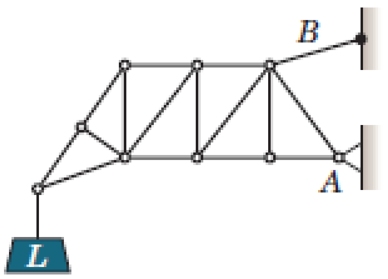
3/C Draw a complete and correct free-body diagram of each of the bodies designated in the statements. The weights of the bodies are significant only if the mass is stated. All forces, known and unknown, should be labeled. (*Note*: The sense of some reaction components cannot always be determined without numerical calculation.).

1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.

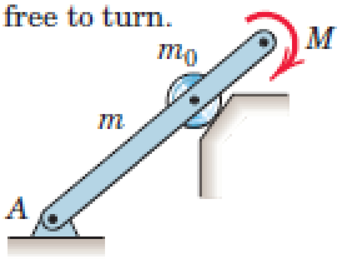
2. Wheel of mass m on verge of being rolled over curb by pull $\mathbf{P}$.



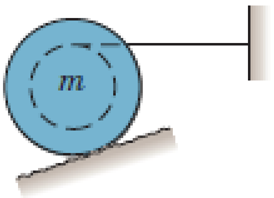
3. Loaded truss supported by pin joint at A and by cable at B.



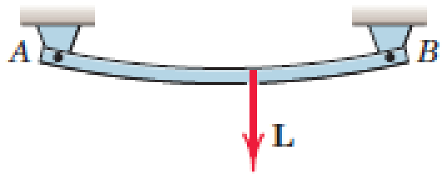
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



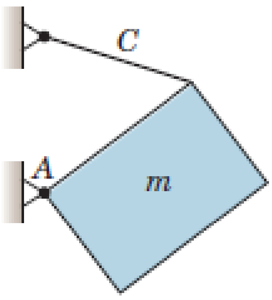
5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.



6. Bar, initially horizontal but deflected under load $\mathbf{L}$. Pinned to rigid support at each end.



7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.

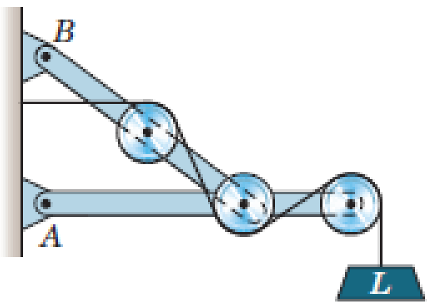


Figure 3/C

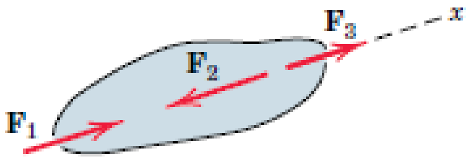
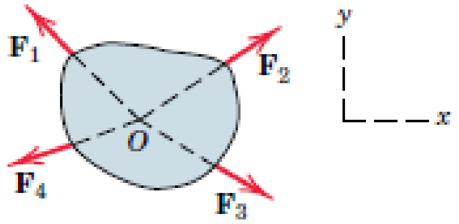
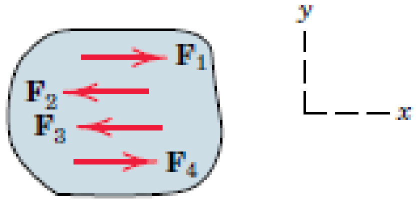
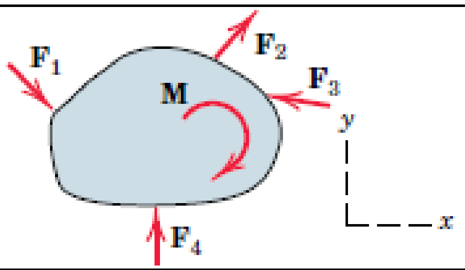
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

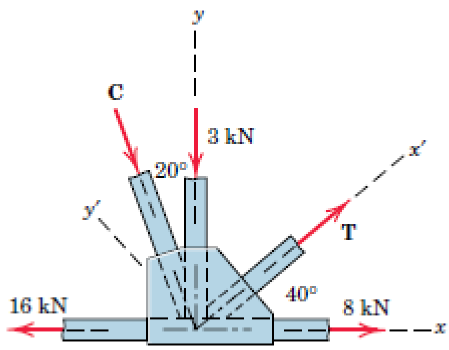
Figure 3/3

3/3 EQUILIBRIUM CONDITIONS

$\Sigma Fx = 0$ $\Sigma Fy = 0$ $\Sigma M_O = 0$

Sample Problem 3/1

Determine the magnitudes of the forces C and T, which, along with the other three forces shown, act on the bridge-truss joint.



Solution. The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium.

Solution I (scalar algebra). For the x-y axes as shown we have

$[\Sigma F_x = 0]$ $\quad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0 \qquad$ (a)

$\quad 0.766T + 0.342C = 8$

$[\Sigma F_y = 0]$ $\quad T\sin 40^\circ - C\cos 20^\circ - 3 = 0 \qquad$ (b)

$\quad 0.643T - 0.940C = 3$

Simultaneous solution of Eqs. (a) and (b) produces

$T = 9.09$ kN $\qquad C = 3.03$ kN $\qquad$ Ans.

Solution II (scalar algebra). To avoid a simultaneous solution, we may use axes x'-y' with the first summation in the y'-direction to eliminate reference to T. Thus,

$[\Sigma F_{y'} = 0]$ $\quad -C\cos 20^\circ - 3\cos 40^\circ - 8\sin 40^\circ + 16\sin 40^\circ = 0$

$\quad C = 3.03$ kN $\qquad$ Ans.

$[\Sigma F_{x'} = 0]$ $\quad T + 8\cos 40^\circ - 16\cos 40^\circ - 3\sin 40^\circ - 3.03\sin 20^\circ = 0$

$\quad T = 9.09$ kN Ans.

Solution III (vector algebra). With unit vectors i and j in the x- and y-directions, the zero summation of forces for equilibrium yields the vector equation

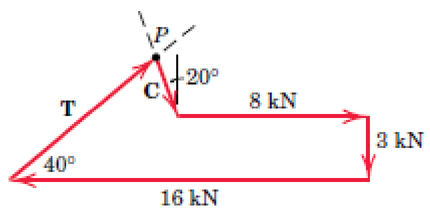
[ΣF=0] $\quad 8\mathbf{i} + (T\cos 40^\circ)\mathbf{i} + (T\sin 40^\circ)\mathbf{j} - 3\mathbf{j} + (C\sin 20^\circ)\mathbf{i} - (C\cos 20^\circ)\mathbf{j} - 16\mathbf{i} = 0$

Equating the coefficients of the i- and j-terms to zero gives

$\quad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$

$\quad T\sin 40^\circ - 3 - C\cos 20^\circ = 0$

which are the same, of course, as Eqs. (a) and (b), which we solved above.



Solution IV (geometric). The polygon representing the zero vector sum of the five forces is shown. Equations (a) and (b) are seen immediately to give the projections of the vectors onto the x- and y-directions. Similarly, projections onto the x'- and y'-directions give the alternative equations in Solution II. A graphical solution is easily obtained. The known vectors are laid and **C** are then drawn to close the polygon. The resulting intersection at point P completes the solution, thus enabling us to measure the magnitudes of **T** and **C** directly from the drawing to whatever degree of accuracy we incorporate in the construction.

Helpful Hints

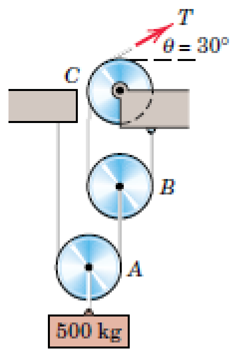
1-Since this is a problem of concurrent forces, no moment equation is necessary.

2- The selection of reference axes to facilitate computation is always an important consideration. Alternatively in this example we could take a set of axes along and normal to the direction of C and employ a force summation normal to C to eliminate it.

3-The known vectors may be added in any order desired, but they must be added before the unknown vectors.

Sample Problem 3/2

Calculate the tension T in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



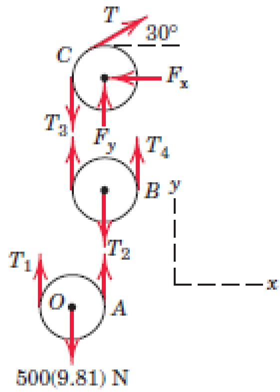
Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A, which includes the only known force.

With the unspecified pulley radius designated by r, the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

$[\Sigma M_O = 0]$ $T_1 r - T_2 r = 0$ $T_1 = T_2$

$[\Sigma F_y = 0]$ $T_1 + T_2 - 500(9.81) = 0$

$2T_1 = 500(9.81)$ $T_1 = T_2 = 2450$ N



From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$T_3 = T_4 = T_2/2 = 1226$ N

For pulley C the angle θ = 30° in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

T = T3 or T = 1226 N Ans.

Equilibrium of the pulley in the x- and y-directions requires

$[\Sigma F_x = 0]$ $1226 \cos 30^\circ - F_x = 0$ $F_x = 1062$ N

$[\Sigma F_y = 0]$ $F_y + 1226 \sin 30^\circ - 1226 = 0$ $F_y = 613$ N

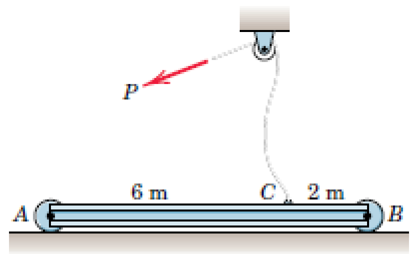
$[F = \sqrt{F_x^2 + F_y^2}]$ $F = \sqrt{(1062)^2 + (613)^2} = 1226$ N Ans.

Helpful Hint

1- Clearly the radius r does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Sample Problem 3/3

The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable at C it is desired to elevate end B to a position 3 m above end A. Determine the required tension P, the reaction at A, and the angle _ made by the beam with the horizontal in the elevated position.



Solution. In constructing the free-body diagram, we note that the reaction on the roller at A and the weight are vertical forces. Consequently, in the absence of other horizontal forces, P must also be vertical. From Sample Problem 3/2 we see immediately that the tension P in the cable equals the tension P applied to the beam at C.

Moment equilibrium about A eliminates force R and gives

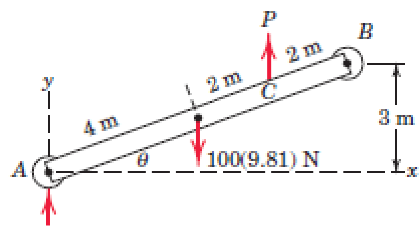
$[\Sigma M_A = 0]$ $P(6 \cos \theta) - 981(4 \cos \theta) = 0$ $P = 654$ N Ans.

Equilibrium of vertical forces requires

$[\Sigma F_y = 0]$ $654 + R$ _ $981 = 0$ R = 327 N Ans.

The angle θ depends only on the specified geometry and is

$\sin \theta = 3/8$ $\theta = 22.0^\circ$



Helpful Hint

1- Clearly the equilibrium of this parallel force system is independent of θ.